Definition 1. *Define an* **improvement ratio** *as the ratio of the improvement of the network overall reliability to the network overall reliability before adding a backup.*

Theorem 2. *Provisioning a backup VNF to two primary VNFs whose reliabilities are among the lowest maximizes the improvement ratio for each case described in Section 3.4.*

We have four cases as described in Sec. 3.4, here we only prove **Thm. 2** for the second case. Note that we can similarly prove **Thm. 2** for other cases as well (in fact, case 1 is simpler, and cases 3 and 4 are based on case 2).

PROOF. Given that two VNFs n_r^i and n_r^j are selected, b as a backup VNF for n_r^i and n_r^j, and n_r^j already has backups. We also assume that the reliability of backup b is a constant. As n_r^j already has backup, we must have computed the reliability of the sub-network N which contains n_r^j already. Then the reliability of the whole network before connecting n_r^i and n_r^j with backup b is $r_{before} = r_1 r_2 ... r_k r_i r_N r_p ... r_q$, and the reliability after adding backup is $r_{after} = r_1 r_2 ... r_k (r_N(1-(1-r_b)(1-r_i)) + r_b r_{N\setminus j}) r_p ... r_q$ where $r_1 r_2 ... r_k$ and $r_p ... r_q$ are the reliability of the sub-networks not selected, and r_N is dependent on r_j. The improvement ratio u is,

$$u = \frac{r_{after} - r_{before}}{r_{before}} = -r_b + \frac{r_b}{r_i r_N}(r_N + r_{N\setminus j}) \quad (6)$$

Let's substitute $r_{N\setminus j}$ with Eq. (3), and let $A = (1 - r_j)\prod_{k=1}^{M}(1 - r_{j_k})$ then calculate the partial derivative with respect to r_i and r_j respectively,

$$u_{r_i} = \frac{\partial u}{\partial r_i} = -\frac{r_b}{r_i^2 r_N}(r_N + r_{N\setminus j}) \quad (7)$$

$$u_{r_j} = \frac{r_b}{r_i}\frac{(A' r_N^2 + 2\frac{\partial r_N}{\partial r_j} A r_N)(\tau - A) - (\tau - A)' A r_N^2}{(\tau - A)^2} \quad (8)$$

As the reliability is always greater than or equal to 0, and $r_i \in [0, 1]$, from Eq. (7) we can easily tell that u decreases monotonically as r_i increases. While Eq. (8) is not that obvious, so we let $u_{r_j} = 0$ to compute the critical point, and get

$$2\frac{\partial r_N}{\partial r_j} = (\frac{A}{\tau - A} + 1)\frac{r_N}{1 - r_j} \quad (9)$$

Solve this partial differential equation, and get

$$r_N = \sqrt{\frac{\frac{\tau}{\prod_{k=1}^{M}(1 - r_{j_k})} - (1 - r_j)}{1 - r_j}} \quad (10)$$

which means that when the equation holds true, we get the critical point. However, $\frac{\tau}{\prod_{k=1}^{M}(1 - r_{j_k})} \gg 1$ since $\tau > 1$, $M \geq 1$ and both $r_N \in [0, 1]$ and $r_j \in [0, 1]$, so this equation can never hold, which means u is a monotone function respect to r_j in its domain. Also we can easily check $u_{r_j=1} < u_{r_j=0}$, so we can come to the same conclusion as for r_i that u decreases monotonically as r_j increases. Therefore, selecting two VNFs with lowest reliability leads to the largest improvement ratio.

4. EVALUATION

Our simulations are conducted over the 14-node NSF network. Each node of the network can provide three types of resources, namely CPU, memory and storage, with the capacity of 3500 units. We assume there are 8 types of functions in the network, and each of the physical node can provide four to six functions. The reliability of each node is randomly distributed within [0.9, 0.99]. The network traffic along each link is carried using *Optical Orthogonal Frequency Division Multiplexing* (OOFDM), because it is a cost-effective technique to achieve Terabit-per-second transmission [10], which is needed to support the huge amount of traffic flow generated by reliable SFC mapping. Each of the links has a spectrum capacity of 12THz with a spacing of 12.5GHz per spectrum slot using OOFDM.

Each SFC request consists of interconnected two to six VNFs. Each VNF demands three types of node resources, and the demand for each kind of resource is uniformly distributed between 0 and 30. Each logical link has a bandwidth demand among {10, 40, 100, 200} Gb/s with equal probability. K is set to 3 for searching shortest paths between two VNFs. For each SFC request, we select the reliability requirement among {95%, 99%, 99.9%}, similar to the ones used by Google Apps [11]. ϵ is set to 0.07 for reliability evaluation. The statistics are the average results.

We first perform simulations to compare the number of SFC requests that can be accepted with different VNF protection strategies and backup selection methods. From Fig. 4, we can see that GREP achieves the best acceptance ratio performance, and in particular, it outperforms SP and DP, both of which adopt the backup selection strategy we propose in Section 3.5, by 13.3% and 21.4%, respectively. To show the effectiveness of our proposed backup selection strategy, we also show the case where we randomly select two VNFs for each iteration to protect. Using JP with random backup selection achieves the same performance as SP with the proposed selection model. We show the efficiency of GREP in terms of the number of backup link used per request (only accepted request are considered) in Fig. 5. Because of the adoption of JP, GREP uses 31% and 14% fewer links compared with the other two methods respectively when the reliability requirement is "three nines" (i.e., 99.9%). The reason is, in JP, two VNFs are protected by one node and if these two VNFs are adjacent or share neighbors, then they can also share logical links that connect the backup VNF. Similar observations can be made when comparing the number of backup VNFs needed as shown in Fig. 6. We can find that GREP requires fewer number of backup VNFs,

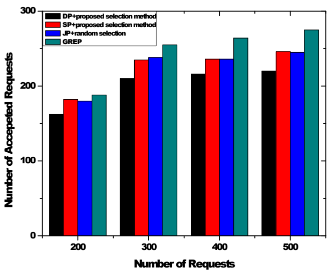

Figure 4: Number of accepted requests under different protection mechanism

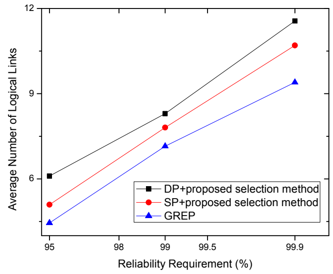

Figure 5: Average backup link number with different reliability requirement

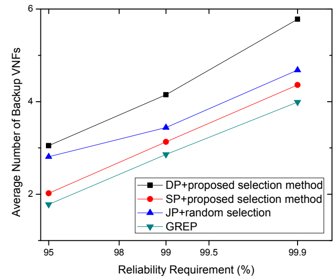

Figure 6: Average backup VNF number with different reliability requirement

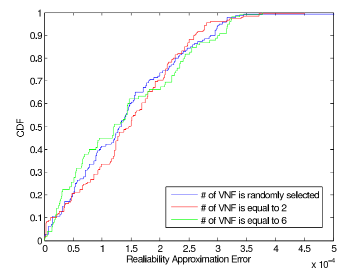

Figure 7: CDF of the error of the proposed approximate reliability calculation method

which implies the potential saving of power consumption using JP (since a fewer backup VNFs means lower static power consumption in PN such as a datacenter). Also, as illustrated in Fig. 6, using our proposed backup selection method merely can save at most 37% of physical nodes. To show the accuracy of our proposed reliability calculation method, we evaluate the *Cumulative Distribution Function* (CDF) of the approximation error of GREP when the number of request is 300 and the reliability threshold is set to 99.9%, as depicted in Fig. 7. We can observe that 98% of the error is smaller than 3.5×10^{-4}, which demonstrates the effectiveness of using GREP to predicting the service reliability.

5. CONCLUSION

In NFV, it is critical to provide effective reliability guarantee with efficient and robust resource allocation in order to support the shared middlebox platform. In this paper, we have proposed GREP for reliable SFC mapping in NFV networks, which can minimize the resources allocated to SFC requests while meeting clients' SLA requirement. We have validated our design through extensive simulations and demonstrated that it can achieve significant performance improvement compared to the traditional protection mechanisms. Meantime, we have shown that GREP is able to evaluate service reliability with a negligible approximation error in polynomial time. As for our future work, we plan to extend the proposed algorithm to (1) jointly optimize the selection of primary and backup mapping nodes, knowing that backups may eventually be needed; (2) share redundancy across multiple SFC requests to further increase resource utilization; (3) take dynamic traffic demands into consideration when devising a backup plan.

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