Pricing Internet Access for Disloyal Users: A Game-Theoretic Analysis

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ABSTRACT

In this paper we investigate the impact of customer loyalty on the price competition between local Internet Service Providers who sell Internet access to end-users. The main contribution of this paper is threefold. First, we develop a repeated game, and show how cooperation between ISPs resulting in higher profits can be enforced through a threat strategy in the presence of customer loyalty. Second, we investigate the case of a differentiated customer population by introducing dual reservation values, and show how it leads to new, pure strategy Nash equilibra for a wide range of demand functions. Third, we develop two novel models for customer loyalty, along with a simulation tool that is capable of demonstrating the impact of the novel models. We argue that our findings can bring us closer to the understanding of economic interactions among ISPs and, at the same time, can motivate researchers to incorporate a finer-grained user behavior model involving customer loyalty in their investigations of such interactions.

Categories and Subject Descriptors

C.4 [**Performance of Systems**]: Modeling techniques; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Economics, Theory, Experimentation

1. INTRODUCTION

Advances in networking technology and affordable service prices are continuing to make Internet access available for billions of customers. To provide end-to-end network connection, Internet Service Providers (ISPs) form a hierarchy that spans from local ISPs who sell access to end-users, through regional ISPs who connect local ISPs to the Internet backbone, to Tier-1 ISPs who form the backbone, and are peering with each other. The economic interactions among service providers of different levels and end-users have been in the focus of interest for several years. Furthermore, these interactions will continue to get special attention, since initiatives like the NSF FIND [1] promote economic incentives as a first-order concern in future network design. Also, decision-makers trying to work out a plausible solution for the recently surfaced net neutrality debate would greatly benefit from an in-depth understanding of economic processes inside the user-ISP hierarchy.

There is broad literature in the area of modeling interactions between ISPs with game-theoretical means [11] [5] [16]. While these papers introduce and analyze complex models for the interaction of ISPs at different levels of the hierarchy, they mostly assume a very simple user behavior model when investigating the market for local ISPs: end-users choose the cheapest provider assuming that the quality of the certain services is the same. This assumption could be plausible in certain scenarios, but it could be misleading if there are loyal customer segments present in the market. On the other hand, economists are well aware of the notion of consumer or brand loyalty, which is very much existing in realistic markets. Practically speaking, a customer is loyal to a brand, when she purchases the product of that brand, even if there are cheaper substitutions on the market. Brand loyalty is rooted in both satisfaction towards a given brand and customers being reluctant to try substitute products. There is existing work dealing with classification of buyers into loyalty groups [17], and a recent study develops and empirically tests a model of antecedents of consumer loyalty towards ISPs [6]. In [12] authors use a game-theoretic framework to prove that if loyalty is an additional product of market share and penetration, customer retention strategies seem to be consequently more efficient for market leaders. An other study [7] analyzes a duopolistic price setting game in which firms have loyal consumer segments, but cannot distinguish them from price sensitive consumers. They demonstrate that consumer loyalty plays an important role in establishing the existence and identity of a price leader.

The latter two papers provide valuable insight to the impact of brand loyalty on certain markets, but also inspire for further investigations. First of all, how does customer loyalty affect a dynamic market of Internet access? Second, [7] only considers perfectly inelastic demand and a single reservation price for the whole customer population. While these two assumptions may hold in certain scenarios, are they valid if considering the Internet access market in developing countries or an economically differentiated Internet user population? Third, are there incentives for cooperative pricing regarding local Internet Sevice Providers in a market where user loyalty is present? And last, is the simple model, which is commonly used in game-theoretic frameworks, a good representation of real-world brand loyalty? Can the real-life behavior of customers (such as sensitivity to the price difference between providers and uncertainty in their decisions) be incorporated into a better user model? We ar-

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Table 1: Payoff matrix for the basic game

	H	M	L
H	$(60, 60)$	$(0, 100)$	$(0, 60)$
M	$(100, 0)$	$(50, 50)$	$(0, 60)$
L	$(60, 0)$	$(60, 0)$	$(30, 30)$

Table 2: Payoff matrix for the brand loyalty game

	H	M	L
H	$(60, 60)$	$(36, 70)$	$(36, 42)$
M	$(70, 36)$	$(50, 50)$	$(30, 42)$
L	$(42, 36)$	$(42, 30)$	$(30, 30)$

gue that finding an answer to these questions can bring us closer to the understanding of economic interactions among ISPs and, at the same time, it can motivate researchers to use a finer-grained user behavior model involving customer loyalty in their investigations.

In this paper we investigate the impact of customer loyalty on the price competition between local ISPs who sell Internet access to end-users, both qualitatively and quantitatively. The main contribution of this paper is threefold. First, we develop a repeated game based on the single-shot game presented in [7], and show how cooperation between ISPs resulting in higher profits can be enforced through a threat strategy in the presence of customer loyalty (Section 3.1). Second, we investigate the case of a differentiated customer population by introducing dual reservation values, and show how it leads to new, pure strategy Nash equilibra. Also, we show how these results hold for a wide range of demand functions (Section 3.2). Third, we develop two novel models for customer loyalty, along with a simulation tool that is capable of demonstrating the impact of the novel loyalty models in price competition among local ISPs (Section 4).

2. MOTIVATION

For illustrating the effect of brand loyalty consider the following game [9]. Suppose there are two restaurants selling pizza in a particular geographic market. Suppose they each consider three possible prices for pizzas: a high price (H), a medium price (M) and a low price (L). The profit per product is known to be \$12, \$10 and \$6 for each firm regardless of the volume of sales. Also let us assume a perfectly inelastic demand function, $D(p) = 10000$, so customers buy 10000 pizzas without regard to its price. The game is similar to the Bertrand game as if the prices of the two firms are different all demand goes to the lower priced firm, and if the prices are equal, firms split the market evenly. It is easy to see that $(p_1^*, p_2^*) = (L, L)$ is the unique Nash equilibrium of the game (see Table 1).

Now, we change the game a little bit, and introduce brand loyalty, such as the firm with the higher price loses some but not all of its customers to the lower priced competitor. Assume that each firm has a loyal customer base that buys 3000 pizzas, and the firms are competing for the remaining demand of 4000 pizzas. In this case the unique Nash equilibrium shifts to $(p_1^*, p_2^*) = (M, M)$ (see Table 2). It turns out that brand loyalty removes the incentive to try to undercut the price of the other firm in order to steal market share.

The game above demonstrates qualitatively how the existence of brand loyalty can affect the outcome of the price competition, by changing the equilibrium point. However, the broad existing literature assessing the pricing competition among Internet access providers (local ISPs) does not take brand (or user) loyalty into consideration resulting in an overly simplified user model. This may lead to imprecise statements regarding equilibrium properties. But what is the particular quantitative impact of user loyalty on local ISP pricing competition? We apply the simple, static loyalty model— used both in the pizza game and [7] to the scenario of multiple local ISPs competing in price to attract customers (users) to show how loyalty could introduce new equilibria, and how cooperation between ISPs can be achieved in the presence of loyalty. Later, in Section 4 we address several shortcomings of the static loyalty model and introduce two novel models, that enable us to incorporate more realistic user behavior to pricing competition. We show the implication of these models to ISP prices and profits by means of simulation.

3. IMPACT OF USER LOYALTY

In this section we construct games involving local ISPs as players, who compete in prices to attract customers. The loyalty model used in these games is similar in nature to the one used in Section 2. There is a fixed loyal user base for each competing service provider, and there is an additional group of potential users, not tied to any firm, seeking the lowest price on the market. We analyze the outcome of the games and show how cooperation between two competing local ISPs can be enforced in the presence of user loyalty (see Section 3.1), and also how a differentiated user population may introduce pure strategy Nash equilibria, which do not exist in a single reservation value scenario. Note, that proofs of propositions are omitted due to space constraints and can be found at [3]. Before getting into the details, we hereby justify our assumptions used in the games throughout this section.

Flat-rate subscriptions. There are repeating patterns in the history of communication technologies, including ordinary mail, the telegraph, the telephone, and the Internet. In particular, the typical story for each service is that quality rises, prices decrease, and usage increases to produce increased total revenues. At the same time, pricing becomes simpler [14]. The schemes that aim to provide differentiated service levels and sophisticated pricing schemes are unlikely to be widely adopted. On the other hand, price and quality differentiation are valuable tools that can provide higher revenues and increase utilization efficiency of a network, and thus in general increase social welfare. It is also shown that flat-rate pricing wastes resources, requires light users to subsidize heavy users, and hinders deployment of broadband access [18]. However, it appears that as communication services become less expensive and are used more frequently, those arguments lose out to customers' desire for simplicity. A success story of the late 1990s was the i-Mode service in Japan, which was the first to offer high-speed mobile Internet access for a flat rate. It succeeded at a time, when other mobile data services were failing, partly because of the pricing scheme. The service is still popular among users, more than 20 percent of NTT DoCoMo customers in Japan have signed up for flat rate mobile internet plans [2]. Furthermore, non-flat rate billing is also resource consuming from a service provider's viewpoint [10]. All of the above, and the fact that most Internet access providers offer flat-rate subscriptions for end-users today, motivates us to assume a flat-rate pricing scheme in our models.

Consumer demand for Internet access. The price elasticity of demand for a particular demand curve is greatly influenced by the degree of necessity or luxury: luxury products tend to have greater elasticity than necessities. The proportion of income required to purchase a service also plays a key role: products requiring a larger portion of the consumer's income tend to have greater elasticity [19]. These two observations suggest that in a developed country, where incomes are high, Internet access is ubiquitous and people tend to lean on the Internet by a great degree (in their work and also during their spare time), almost every household has Internet access, so the demand can be modeled as constant (perfectly inelastic). On the other hand, markets in developing regions are highly price sensitive, since people have lower incomes, and the number of Internet subscriptions would greatly benefit from lower prices. Therefore, the demand for Internet access in such regions can be best modeled as elastic. We use the inelastic model in Section 3.1 to comply with the assumptions of [7], while we investigate both of them in the games of Section 3.2.

Reservation prices of customers. Consumer population is heterogeneous in the sense that certain groups are willing to pay different amounts of money for the same service. In the dream world of ISPs, in which they were able to perfectly identify the reservation price of each customer in the market, they could offer individually differentiated prices, thus squeezing off every cent from the users. Such a perfect identification of reservation prices is not likely in the real world. However, the reservation price of existing customers is generally higher than that of new customers, because existing customers tend to exhibit higher switching costs and also higher brand preference for that product [20]. Furthermore, most of the analytical literature on price discrimination has found that it is optimal to penalize loyals with higher prices than "switchers" [13] [8]. While we do not introduce targeted pricing to our models, we still assume that loyal users inherently tolerate a higher price than "switchers", who are only interested in discount prices. This way, we use dual reservation values in Section 3.2 to represent the heterogeneity of the user population. In Section 3.1 however, we stick to the assumptions of [7] in order to construct a clear extension of that model. In all cases, reservation prices are assumed to be common knowledge.

Although the payoff functions of ISPs are pretty simple across this paper (e.g., marginal cost is set to zero), they are in line with flat-rate pricing, consumer demand elasticity and reservation values discussed above, and thus they suit our needs.

3.1 Incentive to cooperate

Here we present a single-shot game of user loyalty which was introduced in [7]. Later, we extend this game to an infinitely repeated game, and show how a cooperative maximum can be enforced, where the long-term profit of ISPs are higher than that of playing the equilibrium strategy of the stage game in each round.

The stage game. Consider a market with two local ISPs competing in prices for a fixed number of customers. Customers are split into three partitions upon their brand loyalty: the first group consists of l_1 customers who are all loyal to ISP$_1$ in the sense that if ISP$_1$'s price p_1 is less than or equal to a reservation value α, they choose ISP$_1$ as their service provider, otherwise they do not purchase Internet access. The second group consists of l_2 loyal customers of ISP$_2$, while the third group contains n"switchers", who buy service from the cheapest provider, if its price is not greater than α. If the providers announce the same price ($p_1 = p_2 < \alpha$), then half of "the switchers" chooses ISP$_1$ and the other half chooses ISP$_2$. The flow of the game is that ISPs announce their prices simultaneously, then customers make their choices. This game is referred to as G_0.

Note, that though values $l_1 > 0$, $l_2 > 0$ and $\alpha > 0$ are common knowledge, group membership of a given customer cannot be determined, so there is no price discrimination possible. Furthermore, for simplicity we assume a constant unit cost of zero for both firms, and that ISP$_1$ has the larger loyal user base, $l_1 > l_2$.

Given the above and that $p_1 \leq \alpha$ and $p_2 \leq \alpha$, ISP$_1$'s payoff can be expressed as

$$\pi_1(p_1, p_2) = \begin{cases} (l_1 + n)p_1 & p_1 < p_2 \\ (l_1 + 0.5n)p_1 & p_1 = p_2 \\ l_1 p_1 & p_1 > p_2 \end{cases} \quad (1)$$

It can be shown (see [7] and [13]) that this game has a unique Nash equilibrium in mixed strategies. In this case, equilibrium profits are $\pi_1 = l_1\alpha$ and $\pi_2 = \frac{l_2+n}{l_1+n} l_1\alpha$. As it can be noticed, the equilibrium has shifted compared to the simple Bertrand game without consumer loyalty, both parties having a positive payoff in equilibrium.

The repeated game. Now, we extend the previous model, and show that the infinitely repeated G_0 has a subgame perfect equilibrium, which can be enforced by a threat strategy, namely the Nash equilibrium strategy of the stage game G_0.

In the following we construct G_r as the infinitely repeated extension of G_0. Payoff is discounted at step k with a discount factor $\Theta < 1$. The game is continuous at infinity since the discounted payoff in any step is bounded by $\alpha(l_1 + n)$. This way we can use the one-step deviation principle to prove sub-game perfection of a given strategy set.

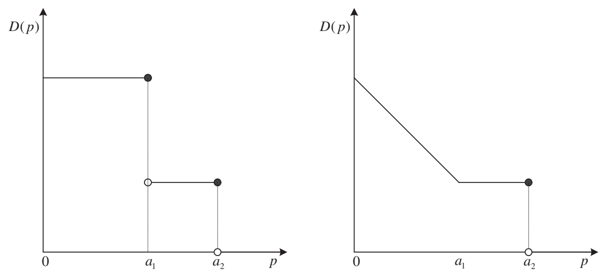
Now, if the two providers cooperate and set their prices equal to the reservation value α, they will share "switchers" equally, in addition to keeping their own loyal users. This way their payoffs (π^{coop}) would be higher than in the equilibrium case (π^{eq}), since $\pi_1^{\text{coop}} = (l_1 + 0.5n)\alpha > \pi_1^{\text{eq}} = l_1\alpha$, and $\pi_2^{\text{coop}} = (l_2 + 0.5n)\alpha > \pi_2^{\text{eq}} = \frac{l_2+n}{l_1+n} l_1\alpha$ if $n > l_1 - 2l_2$. In the cooperative case the joint profit of the two ISPs is the maximum achievable $(n + l_1 + l_2)\alpha$. This cooperation is highly beneficial for both parties. If somehow one ISP tries to grab the whole free market in a single step k, the other ISP can counteract from step $k + 1$ by charging the Nash equilibrium price from G_0 further on, which results in a decreased payoff for the traitor. We show that this Nash reversion assures sub-game perfection for the following strategy profile under the stated conditions.

PROPOSITION 1. *The strategy profile "Cooperate until the other player deviates and then play according to the equilibrium in G_0" is a sub-game perfect Nash equlibrium for the repeated game G_r, if $n > l_1 - 2l_2$ and $\Theta > \frac{1}{2} + \frac{l_1 - \frac{n+l_1}{n+l_2} l_2}{2n}$.*

This means that both the ISP with the smaller and the ISP with the larger loyal user base have an incentive to cooperate in order to maximize their profit on the long run. While explicit cooperation may be illegal, this incentive may lead to discussions between service providers. Note, that a two-ISP setting may seem artificial, it is certainly not, e.g., a large fraction of Internet users in the US can only choose between the local cable and phone company.

3.2 Differentiated reservation prices

Here we construct and analyze single-shot games modeling the price competition between local ISPs fighting for customers with different reservation prices. First, we deal



(a) Inelastic demand (b) Elastic demand

Figure 1: Demand functions for G_1 and G_2

with the case of inelastic demand, and later, we introduce elastic demand.

Inelastic demand. We need to change the single-shot game G_0 a little bit to reflect the duality in reservation prices. We construct G_1 by introducing α_1, the reservation value for "switchers", and α_2, the reservation value for loyal users ($\alpha_1 < \alpha_2$), instead of the single reservation value α. Thus the demand function for G_1 is the following:

$$D(p) = \begin{cases} n + \sum_{j=1}^{N} l_j & 0 \le p \le \alpha_1 \\ \sum_{j=1}^{N} l_j & \alpha_1 < p \le \alpha_2 \\ 0 & p > \alpha_2 \end{cases} \quad (2)$$

where p is the price charged to users and N is the number of competing ISPs. The demand function can be seen in Figure 1(a). From that, we can define the payoff function $\Pi_i(p_i)$ of ISP$_i$, which has a form of

$$\Pi_i(p) = \begin{cases} p_i \left(l_i + \frac{n}{m}\right) & p_i = \min_j p_j \le \alpha_1 \\ p_i l_i & \min_j p_j < p_i \le \alpha_2 \\ 0 & p_i > \alpha_2 \end{cases} \quad (3)$$

where m is the number of ISPs charging the same minimum price, therefore sharing "switchers" equally.

PROPOSITION 2. *Consider G_1 with two players ($N = 2$). Let us define $A = n\alpha_1$ and $B_i = (\alpha_2 - \alpha_1)l_i$ for $i = 1, 2$.*

1. *$(p_1, p_2) = (\alpha_2, \alpha_2)$ is a pure strategy Nash equilibrium, if $A < B_1$ and $A < B_2$;*
2. *$(p_1, p_2) = (\alpha_2, \alpha_1)$ is a pure strategy Nash equilibrium, if $A < B_1$ and $A > B_2$;*
3. *$(p_1, p_2) = (\alpha_1, \alpha_2)$ is a pure strategy Nash equilibrium, if $A > B_1$ and $A < B_2$;*
4. *There is no pure strategy Nash equilibrium if $A > B_1$ and $A > B_2$.*

Elastic demand. A common model used for elastic demand is a linear demand function [19]. We construct the game G_2, by substituting the demand function in G_1 with the one in Figure 1(b):

$$D(p) = \begin{cases} (\alpha_1 - p) + \sum_{j=1}^{N} l_j & 0 \le p \le \alpha_1 \\ \sum_{j=1}^{N} l_j & \alpha_1 < p \le \alpha_2 \\ 0 & p > \alpha_2 \end{cases} \quad (4)$$

From that, we can define the payoff function $\Pi_i(p_i)$ of ISP$_i$, which has a form of

$$\Pi_i(p) = \begin{cases} p_i \left(l_i + \frac{\alpha_1 - p_i}{m}\right) & p_i = \min_j p_j \le \alpha_1 \\ p_i l_i & \min_j p_j < p_i \le \alpha_2 \text{ or} \\ & \alpha_1 < p_i \le \alpha_2 \\ 0 & p_i > \alpha_2 \end{cases} \quad (5)$$

where m is the number of ISPs charging the same minimum price. The equilibrium properties of G_2 are as follows.

PROPOSITION 3. *Consider G_2 with two players ($N = 2$). Let $p_{max} = \text{argmax}_{p \in [0,\alpha_1]} p_i \left(l_i + \frac{\alpha_1 - p_i}{m}\right)$. Let us define $A = (\alpha_1 - p_{max})p_{max}$ and $B_i = (\alpha_2 - p_{max})l_i$ for $i = 1, 2$.*

1. *$(p_1, p_2) = (\alpha_2, \alpha_2)$ is a pure strategy Nash equilibrium, if $A < B_1$ and $A < B_2$;*
2. *$(p_1, p_2) = (\alpha_2, p_{max})$ is a pure strategy Nash equilibrium, if $A < B_1$ and $A > B_2$;*
3. *$(p_1, p_2) = (p_{max}, \alpha_2)$ is a pure strategy Nash equilibrium, if $A > B_1$ and $A < B_2$;*
4. *There is no pure strategy Nash equilibrium if $A > B_1$ and $A > B_2$.*

The logic behind Propositions 2 and 3 is the following. If an ISP has a large loyal user-base, and the ISP can charge them a price high enough, it does not have to deal with disloyal users, since the difference between the profit at price α_2 with loyal users only, is larger than the profit at any price below the α_1 threshold with both loyal and all disloyal users. So if the loyal user population has a high enough reservation value (α_2), their provider can milk them, and it is not interested in undercutting other ISPs to grab "switchers". Note, that we can generalize Proposition 3 to multiple service providers and any reasonable demand function $D(p) = f(p) + \sum_{j=1}^{N} l_j$. Please consult [3] for details.

4. MODELING USER LOYALTY

When the payoff functions of ISPs are known, game theory can be used to find the optimal strategy to maximize profit. But what if you do *not* know the payoff in advance? In reality, an ISP can hardly ever know it exactly. In the simplest model, the profit of an ISP is the product of the demand for its service and the access price. The biggest problem here is modeling the demand, because it depends on many factors, not only on the given access price. It also depends on the price of the competitors, the quality of the given service, and even on human factors. Unfortunately, it is very difficult to incorporate all these factors in a closed form demand function, and hence, it is very difficult to analyze them with game-theoretic tools. Instead, we introduce two extended user loyalty models, which capture important aspects of the nature of demand, and assess the impacts of the models in a simulator. We argue that although intuitively, user loyalty has a crucial effect on demand and profit of ISPs, this subject has not been addressed properly so far in the literature. Through simulations, we show how an ISP can set its price to reach maximum profit in the presence of user loyalty, and also, exactly how much an ISP can benefit from a loyal user base.

4.1 Two Models

While the games in Section 3 have applied a very simple user loyalty model, such a model may not capture the real world characteristics of a pricing competition. The aim of this section is to present two novel approaches for modeling the overall loyalty of a user population. Both models are constructed for using in a repeated price setting scenario, where service providers repeatedly (but simultaneously) set their access prices, e.g., monthly, trying to attract customers. The first model incorporates the price difference among ISPs, while the second model introduces uncertainty in human decisions to user loyalty.

Dealing with price difference – the deterministic model. One logical improvement in loyalty modeling is to determine the amount of "switchers" (the change in the demand) in a single step based on the relative price difference between their current ISP and other ISPs. The justification of this method is that switching providers comes together with some cost to the user (e.g., terminating its current contract, leasing a new access device, etc.), so it is only worth it if the price difference is large enough. Since we focus on the price competition among ISPs where users are not players in a game-theoretic sense, such a factor can only be introduced on a per ISP basis. We achieve this by calculating the number of "switchers" proportional to the price difference between their current provider and the minimum-priced provider(s). Furthermore, because of the time and administration demand on an ISP for terminating the contract of a huge user population (and also on the newly selected ISP for contracting the same amount), there is a hard constraint on the number of "switchers" at a single step. To model this constraint, we have introduced a threshold to limit the number of migrating users.

Based on the above, for a given service provider ISP_i, the number of users it loses to or gains from other providers in round k is defined as

$$\Delta U_i^{(k)} = U_i^{(k-1)} \max\left(\min\left(\frac{\min_j p_j^{(k)} - p_i^{(k)}}{\min_j p_j^{(k)} + p_i^{(k)}}, L_i\right), -L_i\right),$$

where $U_i^{(k-1)}$ is the number of users associated with ISP_i in round $k-1$, $p_i^{(k)}$ is the access price charged by ISP_i in round k, and $L_i \in [0,1]$ represents the administration constraint of ISP_i, and it is a simulation parameter. Note that the number of users in the system (across all ISPs) is modeled as constant, and is normalized to 1.

Dealing with human uncertainty – the stochastic model. In an attempt to cope with uncertainty in human decisions, we reach back to the concept of individual loyalty. We describe a user's individual loyalty by a random variable X with a cumulative density function of $F(x)$. Since ISPs have a large number of users, we then apply the well-known *Central Limit Theorem* to individual loyalty variables to get the loyalty variable of the whole user base of an ISP. Let $X_1, \ldots, X_n$ be identically distributed, independent random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma$ for $1 \le i \le n$, where n is the number of users. $S_n = X_1 + \ldots + X_n$ is the sum of those random variables. Then for a large n, $E(S_n) = n\mu$ and $Var(S_n) = \sqrt{n}\sigma$. Furthermore, $\lim_{n\to\infty} F(s_n) = \Phi\left(\frac{s_n - n\mu}{\sqrt{n}\sigma}\right)$. If the third central moment $E((X_i -)^3)$ exists and is finite, then the speed of convergence is at least on the order of $\frac{1}{\sqrt{n}}$ (see *Berry-Essen theorem* [15]).

We define the loyalty of a user population, S, as the sum of random variables representing individual loyalty, denoted by S_n above. Since $X_1, \ldots, X_n$ have to be iid for the theorem to hold, we make the assumption that individual users' loyalty do not affect each other, rather it is a congenital quality. Also the user population under observation should be relatively big, as we need n to be a fairly large number for the convergence to take effect. On the other hand, we can model an individual user's loyalty with *any* proper probability distribution. We also keep the administration constraint L_i and the dependence on the price difference between the respective ISP and other ISPs. The number of "switchers" for an ISP_i at step k is calculated as following:

$$\Delta U_i^{(k)} = \max\left(\min\left(S_i^{(k)}, L_i\right), -L_i\right), \text{where}$$

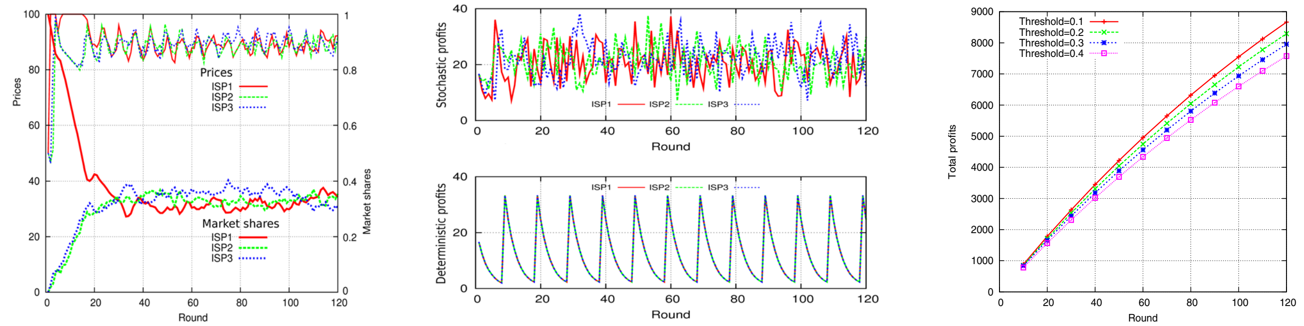
$S_i^{(k)} \sim N\left(U_i^{(k-1)}\mu, \sqrt{U_i^{(k-1)}}\sigma\right)$ and $\mu = \frac{\min_j p_j^{(k)} - p_i^{(k)}}{\min_j p_j^{(k)} + p_i^{(k)}}$ and σ is a simulation parameter.

4.2 Simulation Results

We have developed a simulator to study the impact of the novel loyalty models. Simulations analyze the behavior of competing local ISPs. Each ISP has some end users, a share of the market. We suppose that the overall demand function for their services is constant: no users enter or leave the market. This model is relevant to a saturated market, where everybody can afford to have Internet connectivity, and internet connectivity is a must (see Section 3 for details). We suppose that the user market is infinitely dividable among the fixed number of ISPs. The total end user market is normalized to 1, e.g., if there are 2 ISPs with equal market share, then both of them have a market share of 0.5. ISPs compete for customers by setting their access prices in each round. The price scheme used is flat rate and we assume a homogeneous user population (single, common reservation price). The lowest price an ISP can set for a round is 0, while the highest price is 100, which corresponds to the reservation value common to all users. Before the first round, initial market shares are set. In each round, end users may migrate from their respective provider with regard to the applied loyalty model. "Switchers" choose the cheapest ISP. For simplicity we assume that if there are two minimally-priced ISPs, half of the migrating users joins one and the other half joins the other one. In each round the ISP's try to maximize their instant profit. Each of the ISP's uses the same simple and greedy strategy. They suppose that the prices offered last by their competition stay the same for the next round. With this in mind, they calculate their projected market share change and profit by probing all possible prices they can set, and finally, they choose the price that would maximize their profit in the next round and then play it. The results presented in this section are only intended to flash some interesting issues concerning the impact of user loyalty on the pricing competition of ISPs in different scenarios. For a more comprehensive analysis on simulations for local ISP competition please refer to [3].

Initial market shares. The first interesting results have been produced using the stochastic loyalty model. The initial market share of ISP_1 was set to 1.0 (total market), and the market shares of all other ISPs was set to 0. Figure 2(a) show the prices and market shares (also corresponding to instant profits) of 3 competing ISPs in time. What can be seen is that lower market share ISPs start grabbing the market from ISP_1 by setting lower prices. This makes the higher share ISP lower its prices as well, until a state close to equal market shares is reached. The profit chart teaches us exactly what is logical intuitively. If you have a large loyal user base, you make the highest profit by setting high prices. Small companies make the highest profit by setting discount prices, but can only dream of the profits of the large players. The larger your loyal user base is, the more instant profit you make.

Deterministic vs. stochastic loyalty model. Now let us take a look at the game, when the initial market shares are equal, but let us use different loyalty models. If we compare the profits of 3 ISPs with different loyalty models, it can be noticed that in the deterministic model all the ISPs get the same profit, while in the other case, stochasticity results in slightly different profits for different providers (see Figure 2(b)). The deterministic curve has a periodic shape. The explanation behind this is that it is worth gradually lowering your access price to undercut others and steal market share. Of course there is a certain price under which the respective ISP would get less money by further undercutting than by "milking" its currently associated customers. When this



(a) Prices and user bases–stochastic model

(b) Profits in time

(c) Overall cumulated profits of 2 ISPs

Figure 2: Simulation results

value is reached, the ISPs set the price significantly higher. Note, that all ISPs think the same (because of the simple strategy applied, see above), so their prices (and profits and market shares) will be exactly the same across time (that is why the curves overlap). On the other hand, when user base changes are random, ISPs do not follow exactly the same lines of thought, so they get different payoffs. It is worth mentioning that profits fluctuate around the even shares.

Level of loyalty in the user population. Another impact of loyalty affects the overall profit of ISPs. Here, we discount profits in time with a discount factor of 0.995. In Figure 2(c) the overall sum profit of 2 ISPs can be seen in time. Different lines denote different levels of loyalty: the higher the threshold, the more users can switch providers at a single step (see Section 4.1). Results show that the higher the level of loyalty (i.e., the lower the threshold), the higher overall sum profit can be achieved over time, implying that ISPs are interested in users being loyal to them, since stronger loyalty results in higher overall profits.

Discussion. Our simulation results are consistent with the findings of recent empirical surveys on loyalty in the wired and wireless ISP market [4]. First, they point out the existence of a truly loyal customer segment ($\tilde{3}8\%$) which tolerates higher prices, and is likely to pay for new services and looks for a long-term business relationship. On the other hand, there are “high risk” customers ($\tilde{3}0\%$)who are willing to switch providers at the earliest opportunity, and are driven by both lower prices and (congenital) behavior. Second, while 78% of the customers are “satisfied” with the service they get, only the above-mentioned 38% are truly loyal, hence there is more to loyalty than being satisfied (behavioral patterns). Third, they show that ISPs with the most loyal customers (“loyalty leaders”) can expect significantly larger revenues, faster growth and higher stock price performance than their competitors. We believe that these results justify the importance of user loyalty modeling with regard to pricing Internet access.

5. CONCLUSION

In this paper we have studied multiple facets of the impact of customer loyalty on the price competition of geographically co-located ISPs. We first showed how cooperation between ISPs resulting in higher profits can be enforced in the presence of customer loyalty. Second, we investigated the case of a differentiated customer population by introducing dual reservation values, and show how it leads to new, pure strategy Nash equilibra for a wide range of demand functions. Third, we developed two novel models for customer loyalty, along with a simulation tool that is capable of demonstrating the impact of the novel models, and showed the numerical effect of user loyalty on the price competition through these more realistic models. Of course, there is remaining work to be done: we would like to incorporate our novel loyalty models into a game-theoretic framework, furthermore, we would like to investigate the case of targeted pricing on the Internet access market.

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